

# Sakai-Sugimoto model, tachyon condensation and chiral symmetry breaking

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**ABSTRACT:** We modify the Sakai-Sugimoto model of chiral symmetry breaking to take into account the open string tachyon which stretches between the flavour  $D8$ -branes and  $\overline{D8}$ -branes. There are several reasons of consistency for doing this: (i) Even if it might be reasonable to ignore the tachyon in the ultraviolet where the flavour branes and antibranes are well separated and the tachyon is small, it is likely to condense and acquire large values in the infrared where the branes meet. This takes the system far away from the perturbatively stable minimum of the Sakai-Sugimoto model; (ii) The bifundamental coupling of the tachyon to fermions of opposite chirality makes it a suitable candidate for the quark mass and chiral condensate parameters. We show that the modified Sakai-Sugimoto model with the tachyon present has a classical solution satisfying all the desired consistency properties. In this solution chiral symmetry breaking coincides with tachyon condensation. We identify the parameters corresponding to the quark mass and the chiral condensate and also briefly discuss the mesonic spectra.

**KEYWORDS:** Gauge-gravity correspondence, Intersecting branes models, Chiral Lagrangians.

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## 1. Introduction

The study of connections between gauge theory and string theory in the last decade, following the AdS/CFT conjecture [1, 2], has led to the development of new tools for investigating strong coupling phenomena in gauge theories [3–7]. These ‘holographic methods’ have been used with surprising success in qualitative studies of confinement and chiral symmetry breaking in realistic QCD-like gauge theories, although application to real QCD, which requires quantizing strings moving on highly curved spaces in the presence of RR backgrounds, is still beyond the currently available tools.

In the context of these holographic methods, a subject that has received a lot of attention recently is that of chiral symmetry breaking in QCD-like gauge theories. In holographic models of gauge theories, the Yang-Mills fields arise from massless open string fluctuations of a stack of ‘colour’ branes. The near horizon, strong coupling limit of a large number  $N_c$  of colour branes has a dual description in terms of a classical gravity theory. Flavour degrees of freedom are introduced in this setting as the fermionic open string fluctuations between the colour branes and an additional set of ‘flavour’ branes [8–13]. In the probe approximation in which the number of flavour branes,  $N_f$ , remains finite as  $N_c \rightarrow \infty$ , the backreaction of the flavour branes on the background geometry can be neglected and various phenomena associated with flavour physics studied as classical effects in the background geometry.

The model of Sakai and Sugimoto [14], which is based on this scenario, has been very successful in reproducing many of the qualitative features of non-abelian chiral symmetry

breaking in QCD. In this model, chiral symmetry breaking has a nice geometrical picture. In the ultraviolet, chiral symmetry arises on flavour  $D8$ -branes and  $\overline{D8}$ -branes, which are located at well-separated points on a circle, while they are extended along the remaining eight spatial directions, including the holographic radial direction. Chiral symmetry breaking in the infrared is signalled by a smooth joining of the flavour branes and antibranes at some point in the bulk. At finite temperatures, chiral symmetry is restored at or above the deconfinement transition [16–18].

Despite its many qualitative and, remarkably, some quantitative successes [14, 15, 19–23], the Sakai-Sugimoto model has some deficiencies. As has been pointed out by many authors,<sup>1</sup> this model does not have parameters associated either with the chiral condensate or with quark bare mass. In addition, the model ignores the open string tachyon between  $D8$ -brane and  $\overline{D8}$ -brane, which may be reasonable in the ultraviolet where the branes and antibranes are well separated, but is not so in the infrared where the branes join.<sup>2</sup> In this region one would expect the tachyon to condense. Since the tachyon field takes an infinitely large value in the true ground state,<sup>3</sup> the perturbative stability argument given in [14], valid for small fluctuations of the tachyon field near the local minimum at the origin, does not apply.

Recently, it was suggested in [27] that tachyon condensation on a coincident brane-antibrane configuration describes the physics of chiral symmetry breaking in a better and more complete way. Unfortunately in this scenario one loses the nice geometric picture of the Sakai-Sugimoto model for non-abelian chiral symmetry breaking. The aim of the present work is to develop a model which retains the nice features of the Sakai-Sugimoto model while overcoming its deficiencies. We argue that this can be done by taking into account the open string tachyon that stretches between separated  $D8$ -branes and  $\overline{D8}$ -branes. We will show that in our model, chiral symmetry breaking, which is signalled by joining of branes and antibranes, is accompanied by tachyon condensation, since the tachyon field takes large values only in the region where the branes and antibranes join. Furthermore, the tachyon profile provides the necessary parameters to describe both the quark mass and the chiral condensate.

The organization of this paper is as follows. In the next section we will briefly review the essential features of the Sakai-Sugimoto model. In section 3 we describe the modification in this model required to include the open string tachyon between the  $D8$ -branes and  $\overline{D8}$ -branes. We compute the contribution to the bulk energy momentum tensor of this system and verify that the backreaction is small everywhere. In this section we also obtain the classical solution for the brane profile and the tachyon and identify the parameters associated with the quark mass and the chiral condensate. Mesonic fluctuations around this classical solution are briefly discussed in section 4. We end with a discussion in section 5.

As this work was nearing completion, the paper [28] appeared on the archive which also discusses similar issues.

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<sup>1</sup>See, for example [16, 24, 17, 25].

<sup>2</sup>Actually, even in the ultraviolet region this is not so straightforward, see the discussion in [17].

<sup>3</sup>For a recent review of this subject, see [26].

## 2. The Sakai-Sugimoto model

The Yang-Mills part of this model is provided by the near horizon limit of a set of  $N_c$  overlapping  $D4$ -branes, filling the  $(3 + 1)$ -dimensional space-time directions  $x^\mu$  ( $\mu = 1, 2, 3$  and  $0$ ) and wrapping a circle in the  $x^4$  direction of radius  $R_k$ , with antiperiodic boundary condition for fermions, which gives masses to all fermions at the tree level (and scalars at one-loop level) and breaks all supersymmetries. At low energies, the theory on the  $D4$ -branes is  $(4 + 1)$ -dimensional pure Yang-Mills with 't Hooft coupling  $\lambda_5 = (2\pi)^2 g_s l_s N_c$  of length dimension. At energies lower than the Kaluza-Klein scale  $1/R_k$ , this reduces to pure Yang-Mills in  $(3 + 1)$  dimensions. This is true in the weak coupling regime,  $\lambda_5 \ll R_k$ , in which the dimensionally transmuted scale developed in the effective Yang-Mills theory in  $(3 + 1)$  dimensions is much smaller than the Kaluza-Klein scale, which is the high energy cut-off for the effective theory. In the strong coupling regime,  $\lambda_5 \gg R_k$ , in which the dual gravity description is reliable, these two scales are similar. Therefore in this regime there is no separation between the masses of glueballs and Kaluza-Klein states. This is one of the reasons why the gravity regime does not describe real QCD, but the belief is that qualitative features of QCD like confinement and chiral symmetry breaking, which are easy to study in the strong coupling regime, survive tuning of the dimensionless parameter  $\lambda_5/R_k$  to low values.

Sakai and Sugimoto introduced flavours in this setting by placing a stack of  $N_f$  overlapping  $D8$ -branes at the point  $x_L^4$  and  $N_f$   $\overline{D8}$ -branes at the point  $x_R^4$  on the thermal circle. Massless open strings between  $D4$ -branes and  $D8$ -branes, which are confined to the  $(3 + 1)$ -dimensional space-time intersection of the branes, provide  $N_f$  left-handed flavours. Similarly, massless open strings between  $D4$ -branes and  $\overline{D8}$ -branes provide an equal number of right-handed flavours, leading to a global  $U(N_f)_L \times U(N_f)_R$  chiral symmetry. This global chiral symmetry is visible on the  $D8$  and  $\overline{D8}$ -branes as chiral gauge symmetry.

In the large  $N_c$  and strong coupling limit the appropriate description of the wrapped  $D4$ -branes is given by the dual background geometry. This background solution can be obtained from the type IIA sugra solution for non-extremal  $D4$ -branes by a wick rotation of one of the four noncompact directions which the  $D4$ -branes fill, in addition to wrapping the compact (temperature) direction. In the near horizon limit, it is given by [4, 29]

$$\begin{aligned}
 ds^2 &= \left(\frac{U}{R}\right)^{3/2} (\eta_{\mu\nu} dx^\mu dx^\nu + f(U) (dx^4)^2) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \\
 e^\phi &= g_s \left(\frac{U}{R}\right)^{3/4}, \quad F_4 = \frac{2\pi N_c}{V_4} \epsilon_4, \quad f(U) = 1 - \frac{U_k^3}{U^3}, \quad (2.1)
 \end{aligned}$$

where  $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$  and  $U_k$  is a constant parameter of the solution.  $R$  is related to the 5-d Yang-Mills coupling by  $R^3 = \frac{\lambda_5 \alpha'}{4\pi}$ . Also,  $d\Omega_4$ ,  $\epsilon_4$  and  $V_4 = 8\pi^2/3$  are respectively the line element, the volume form and the volume of a unit  $S^4$ .

The above metric has a conical singularity at  $U = U_k$  in the  $U - x^4$  subspace which can be avoided only if  $x^4$  has a specific periodicity. This condition relates the radius of the

circle in the  $x^4$  direction to the parameters of the background by

$$R_k = \frac{2}{3} \left( \frac{R^3}{U_k} \right)^{\frac{1}{2}} \quad (2.2)$$

For  $\lambda_5 \gg R_k$  the curvature is small everywhere and so the approximation to a classical gravity background is reliable. As discussed in [29], at very large values of  $U$ , the string coupling becomes large and one has to lift the background over to the 11-dimensional M-theory description.

Now consider a set of  $N_f$   $D8-\overline{D8}$ -brane pairs in the above background, placed at points  $x_L^4 = l/2$  and  $x_R^4 = -l/2$  respectively on the circle. If  $N_f$  is kept fixed as the large  $N_c$  limit is taken, the effect of the flavour branes on the background geometry should be small and may be treated in the probe approximation. For the simple case of a single  $D8-\overline{D8}$ -brane pair, the action is

$$S = -\mu_8 \int d^9 \sigma e^{-\phi} \left( \sqrt{-\det A_L} + \sqrt{-\det A_R} \right),$$

where  $\mu_8 = 1/(2\pi)^8 l_s^9$  and  $(A_{L,R})_{ab} = g_{MN} \partial_a x_{L,R}^M \partial_b x_{L,R}^N$  is the induced metric on the brane. The indices  $a, b$  run over the world-volume directions of the branes while the indices  $M, N$  run over the background ten-dimensional space-time directions. Using the static gauge and assuming  $l$  depends on  $U$  only, the action becomes

$$S = -\mathcal{T}_8 V_4 \int d^4 x \int dU \left( \frac{U}{R} \right)^{-3/4} U^4 \left( \sqrt{D_L} + \sqrt{D_R} \right),$$

where  $\mathcal{T}_8 = \mu_8/g_s$  is the  $D8$ -brane tension and

$$D_L = D_R \equiv D = f(U)^{-1} \left( \frac{U}{R} \right)^{-3/2} + f(U) \left( \frac{U}{R} \right)^{3/2} \frac{l'(U)^2}{4}. \quad (2.3)$$

Here and in the following a prime denotes derivative with respect to  $U$ .

In the above setting chiral symmetry breaking has a geometrical description. It is signaled by the brane-antibrane meeting at an interior point  $U \geq U_k$ , even when they are well separated asymptotically. This is because in the background geometry (2.1) the branes have nowhere to end and hence they must meet. This can also be seen by explicitly solving the equation of motion for  $l(U)$  obtained from the above action. This equation is

$$\left( \frac{\left( \frac{U}{R} \right)^{13/4} f(U)}{\sqrt{D}} \frac{f(U)}{4} \left( \frac{U}{R} \right)^{3/2} l'(U) \right)' = 0, \quad (2.4)$$

which has the solution

$$\frac{l(U)}{2} = U_0^4 f(U_0)^{1/2} \int_{U_0}^U dy \frac{f(y)^{-1} \left( \frac{y}{R} \right)^{-3/2}}{\sqrt{y^8 f(y) - U_0^8 f(U_0)}}. \quad (2.5)$$

The branes meet at the point  $U = U_0$ , so  $l(U_0) = 0$ . Moreover, the solution determines the asymptotic separation  $l_0$  of the branes in terms of  $U_0$ . The case in which there is maximum

separation between the brane and antibrane,  $l_0 = \pi R_k$ , is special since in this case  $l(U)$  is independent of  $U$ .

In the generic case, the brane-antibrane system looks like a single brane, coming in from the asymptotic region, turning around near  $U = U_0$  and returning back to the position of the other brane in the asymptotic region. Expanding around the point  $U = U_0$ , we get from (2.5)

$$\frac{l(U)}{2} = \frac{R^{3/2}}{U_0 \sqrt{f(U_0)}} \frac{(U - U_0)^{1/2}}{\sqrt{3 + 5f(U_0)}} [1 + O(U - U_0)]. \quad (2.6)$$

We see that  $l'(U) \sim (U - U_0)^{-1/2}$  diverges near the turning point of the brane profile, as required by a smooth joining of the brane with the anti-brane.

### 3. Sakai-Sugimoto with tachyon

The effective field theory describing the dynamics of a brane-antibrane pair<sup>4</sup> with the tachyon included has been discussed in [31, 32]. The simplest case occurs when the brane and antibrane are on top of each other since in this case all the transverse scalars are set to zero. This is the situation considered in [27]. However, in this configuration one loses the nice geometrical picture of chiral symmetry breaking of the Sakai-Sugimoto model. Since we would like to retain this geometrical picture, we must consider the case when the brane and antibrane are separated in the compact  $x^4$  direction. This requires construction of an effective tachyon action on a brane-antibrane pair, taking into account the transverse scalars. Such an effective action with the brane and antibrane separated along a noncompact direction has been proposed in [31, 32]. A generalization of this action to the present case when the brane and antibrane are separated along a periodic direction is not known. However, for small separation compared to the radius of the circle, the action in [32] should provide a reasonable approximation. In the following we will assume this to be the case. Then, the effective tachyon action for  $l(U) \ll R_k$  is

$$S = - \int d^9 \sigma V(T, l) e^{-\phi} \left( \sqrt{-\det A_L} + \sqrt{-\det A_R} \right), \quad (3.1)$$

$$(A_i)_{ab} = \left( g_{MN} - \frac{T^2 l^2}{Q} g_{M4} g_{4N} \right) \partial_a x_i^M \partial_b x_i^N + F_{ab}^i + \frac{1}{2Q} \left( (D_a \tau (D_b \tau)^* + (D_a \tau)^* D_b \tau) \right. \\ \left. + i l (g_{a4} + \partial_a x_i^4 g_{44}) (\tau (D_b \tau)^* - \tau^* D_b \tau) + i l (\tau (D_a \tau)^* - \tau^* D_a \tau) (g_{4b} - \partial_b x_i^4 g_{44}) \right),$$

where

$$Q = 1 + T^2 l^2 g_{44}, \quad D_a \tau = \partial_a \tau - i(A_{L,a} - A_{R,a})\tau, \quad V(T, l) = g_s V(T) \sqrt{Q}. \quad (3.2)$$

$T = |\tau|$ ,  $i = L, R$  and we have used the fact that the background does not depend on  $x^4$ . Also, in writing the above we are using the convention  $2\pi\alpha' = 1$ .<sup>5</sup>

<sup>4</sup>For simplicity, we will continue to discuss the case of a single flavour, namely one brane-antibrane pair. Generalization to the multi-flavour case can be done using the symmetrized trace prescription of [30].

<sup>5</sup>The complete action also includes Chern-Simons (CS) couplings of the gauge fields and the tachyon to the RR background sourced by the  $D4$ -branes. These will not be needed in the following analysis and hence have not been included here.

The potential  $V(T)$  depends only on the modulus  $T$  of the complex tachyon  $\tau$ . It is believed that  $V(T)$  satisfies the following general properties [26]:

- $V(T)$  has a maximum at  $T = 0$  with  $V(0) = \mathcal{T}_8$ .
- The normalization of  $V(T)$  is fixed by the requirement that the vortex solution on the brane-antibrane system produce the correct relation between  $Dp$  and  $D(p-2)$ -brane tensions.
- In flat space for brane-antibrane on top of each other (i.e. for  $l = 0$ ), the expansion of  $V(T)$  around  $T = 0$  upto terms quadratic in  $T$  gives rise to a tachyon with mass-squared equal to  $-\pi$  in our conventions.
- $V(T)$  has a minimum at  $T = \infty$  where it vanishes.

There are several proposals for  $V(T)$  which satisfy these requirements [26], although no rigorous derivation exists. In view of this, in the following analysis we will avoid using any specific expression for  $V(T)$ , except when needed for explicit numerical calculations. It will, however, be necessary for us to specify the asymptotic form of the potential for large  $T$ . We will assume that in our parametrization this behaviour is given by  $V(T) \sim e^{-cT}$  where  $c$  is a positive constant. A potential satisfying this property, in addition to the properties listed above is [33–35]

$$V(T) = \frac{\mathcal{T}_8}{\cosh\sqrt{\pi}T}. \tag{3.3}$$

### 3.1 Backreaction of the flavour branes

Let us now first discuss the backreaction on the background geometry. For this we need to compute the contribution of the flavour brane-antibrane system to the ten-dimensional bulk energy momentum tensor. Our starting point is the action (3.1). The energy momentum tensor is obtained from it by calculating its functional derivative w.r.t. the background ten-dimensional metric  $g_{MN}$ . The precise relation is  $T^{MN} = 2/\sqrt{-\det g} \delta S/\delta g_{MN}$ . We get,

$$\begin{aligned} T_i^{ab} &= -g_s V(T) \sqrt{Q} e^{-\phi} \frac{\sqrt{-\det A_i}}{\sqrt{-\det g}} (A_i^{-1})_S^{ab}, \\ T_i^{a4} &= -g_s V(T) \sqrt{Q} e^{-\phi} \frac{\sqrt{-\det A_i}}{\sqrt{-\det g}} 2 (A_i^{-1})_S^{ab} (\partial_b x_i^4 - T^2 l A_b), \\ T_i^{44} &= -g_s V(T) \frac{1}{\sqrt{Q}} e^{-\phi} \frac{\sqrt{-\det A_i}}{\sqrt{-\det g}} \times \\ &\quad \times \left[ -8T^2 l^2 + (A_i^{-1})^{ba} (T^2 l^2 (g_{ab} + F_{ab}) + \partial_a x_i^4 \partial_b x_i^4 + T^2 l (A_a \partial_b x_i^4 - a \leftrightarrow b)) \right], \end{aligned} \tag{3.4}$$

where  $i = L$  ( $R$ ) denotes the contribution of the  $D8$ -brane ( $\overline{D8}$ -brane) and the subscript ‘ $S$ ’ stands for the symmetric part. Also, we have defined  $A_b \equiv (A_{Lb} - A_{Rb} - \partial_a \theta)$ , where  $\theta$  is the phase of the complex tachyon,  $\tau = T e^{i\theta}$ . It is understood that each of the above expressions must be multiplied by a position space delta-function specifying the location

of the brane in the transverse space where its contribution to the ten-dimensional bulk energy momentum tensor is localized.

Specializing these expressions to the case of the background solution where the gauge fields are set to zero and  $T$  and  $l$  are functions of  $U$  only, we get

$$\begin{aligned}
 T_i^{ab} &= -V(T) \left(\frac{U}{R}\right)^{-3/4} \sqrt{D_T} g^{ab}, \quad a, b \neq U, \\
 T_i^{UU} &= -V(T) \left(\frac{U}{R}\right)^{-3/4} \frac{Q}{\sqrt{D_T}}, \\
 T_i^{44} &= -V(T) \left(\frac{U}{R}\right)^{-3/4} \frac{f^{-1} \left(\frac{U}{R}\right)^{-3/2}}{\sqrt{D_T}} \left( T^2 l^2 + f \left(\frac{U}{R}\right)^{3/2} \frac{l'^2}{4} \right), \quad (3.5)
 \end{aligned}$$

and all other components vanish. The quantity  $D_T$  is defined in (3.7). If  $T$  goes to infinity near the place where the brane and the antibrane meet, all the components of the energy momentum tensor vanish there because  $V(T) \rightarrow 0$  exponentially for large values of  $T$ . Thus the situation is even better than without the tachyon<sup>6</sup> and the flavour contribution to the energy momentum tensor is small everywhere, justifying the probe approximation for a generic configuration.

Recently a detailed calculation of the backreaction of the flavour branes on the geometry in the Sakai-Sugimoto model has been reported in [40]. In this work the calculation has been done for the special configuration in which the branes and antibranes are separated maximally on the circle, i.e.  $l = \pi R_k$ . The authors find that, as expected, in this antipodal case the corrections are indeed small for  $N_f/N_c$  small. It would be interesting to extend their calculation to the generic case with the tachyon present.

### 3.2 Tachyon condensation as chiral symmetry breaking

We will now look for an appropriate classical solution of the brane-antibrane-tachyon system. Let us set the gauge fields and all but the derivatives with respect to  $U$  of  $T$  and  $x_i^4$  to zero. Moreover, we choose  $x_L^4 = l/2$  and  $x_R^4 = -l/2$  so that the separation between the brane and antibrane is  $l$ . In this case, in the static gauge the action (3.1) simplifies to<sup>7</sup>

$$S = -V_4 \int d^4x \int dU V(T) \left(\frac{U}{R}\right)^{-3/4} U^4 \left( \sqrt{D_{L,T}} + \sqrt{D_{R,T}} \right), \quad (3.6)$$

where  $D_{L,T} = D_{R,T} \equiv D_T$  and

$$D_T = f(U)^{-1} \left(\frac{U}{R}\right)^{-3/2} + f(U) \left(\frac{U}{R}\right)^{3/2} \frac{l'(U)^2}{4} + T'(U)^2 + T(U)^2 l(U)^2. \quad (3.7)$$

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<sup>6</sup>In the absence of the tachyon, the energy momentum tensor components in (3.5) blow up near the place where the brane and the antibrane meet. This is, however, not a real singularity since it can be removed by changing the description, for example, to  $U$  as a function of  $l$  instead of the description in terms of  $l(U)$ .

<sup>7</sup>The CS term in the action does not contribute for such configurations.



The equations of motion obtained from this action are

$$\begin{aligned} \left( \frac{U^{\frac{13}{4}}}{\sqrt{D_T}} T'(U) \right)' &= \frac{U^{\frac{13}{4}}}{\sqrt{D_T}} \left[ T(U) l(U)^2 + \frac{V'(T)}{V(T)} (D_T - T'(U)^2) \right], \quad (3.8) \\ \left( \frac{U^{\frac{13}{4}}}{\sqrt{D_T}} \frac{f(U)}{4} \left( \frac{U}{R} \right)^{\frac{3}{2}} l'(U) \right)' &= \frac{U^{\frac{13}{4}}}{\sqrt{D_T}} \left[ T(U)^2 l(U) - \frac{V'(T)}{V(T)} \frac{f(U)}{4} \left( \frac{U}{R} \right)^{\frac{3}{2}} l'(U) T'(U) \right]. \end{aligned} \quad (3.9)$$

Note that the ‘prime’ on  $V(T)$  denotes a derivative w.r.t. its argument  $T$  and not a derivative w.r.t.  $U$ .

This is a complicated set of coupled nonlinear differential equations. To get some insight into the kind of solutions that are possible, we will first analyse the equations for large  $U$  and for  $U$  near the brane-antibrane joining point, where the equations simplify and can be treated analytically. As in the case without the tachyon, we are looking for solutions in which the brane and antibrane have an asymptotic separation  $l_0$ , i.e.  $l(U) \rightarrow l_0$  as  $U \rightarrow \infty$  and they join at some interior point in the bulk, i.e.  $l(U) \rightarrow 0$  at  $U = U_0 > U_k$ . Moreover, we want the tachyon (i) to vanish as  $U \rightarrow \infty$  so that the chiral symmetry is intact in the ultraviolet region and (ii) to go to infinity as  $U$  approaches  $U_0$  so as to reproduce correctly the QCD chiral anomalies [27].

### 3.2.1 Solution for large $U$

We are looking for a solution in which  $l(U)$  approaches a constant  $l_0$  and  $T$  becomes small as  $U \rightarrow \infty$ . Let us first consider the equation (3.8). For small  $T$  one can approximate  $V'/V \sim -\pi T$ .<sup>8</sup> If  $T$  and  $l'$  go to zero sufficiently fast as  $U \rightarrow \infty$  such that to the leading order one might approximate  $D_T \sim \left(\frac{U}{R}\right)^{-3/2}$ , then (3.8) reduces to

$$(U^4 T'(U))' = l_0^2 U^4 T. \quad (3.10)$$

This equation can be solved exactly with the general solution

$$T(U) = \frac{T_+}{U^2} \left( 1 + \frac{1}{l_0 U} \right) e^{-l_0 U} + \frac{T_-}{U^2} \left( 1 - \frac{1}{l_0 U} \right) e^{l_0 U}, \quad (3.11)$$

The solution with the exponential fall off satisfies the approximations under which (3.10) was derived for any large value of  $U$ . The exponentially rising solution will, however, eventually become large and cannot be self consistently used. This is because for sufficiently large  $U$ , there is no consistent solution for  $T$  which grows exponentially or even as a power-law to the original equations (3.8) and (3.9), if we impose the restriction that  $l(U)$  should go to a constant asymptotically. This puts a restriction on the value of  $U$  beyond which the generic solution (3.11) cannot be used. The most restrictive condition comes from the approximation  $D_T \sim \left(\frac{U}{R}\right)^{-3/2}$ . This requires the maximum value,  $U_{\max}$ , to satisfy  $U_{\max}^{5/2} e^{-2l_0 U_{\max}} \gg l_0^2 T_-^2 R^{-3/2}$ . At values of  $U$  much larger than this, only the exponentially falling part provides a consistent solution.

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<sup>8</sup>This follows from the general properties of the potential discussed in section 3.

Even though (3.11) does not represent a truly asymptotic solution, its usefulness lies in the fact that most quantities of interest that involve the tachyon, like pseudoscalar meson masses, receive maximum contribution from intermediate values of  $U$  and hence from this solution. This is because the exponentially falling tachyon potential kills off contribution in the infrared region and the exponentially falling tachyon does so in the ultraviolet region, so the maximum contribution comes from intermediate region. Thus physical quantities are sensitive to both the parameters of this solution. It is natural to associate  $T_-$  with the quark bare mass since this parameter comes with the growing solution and  $T_+$  with the chiral condensate because it is associated with the normalizable solution. More evidence for this will be given in the next section.

The fact that the tachyon takes small values for large  $U$  makes it irrelevant for the leading behaviour of  $l$ , which can be extracted from (3.9) by setting the r.h.s. to zero. The resulting equation is precisely (2.4) with a similar solution

$$l(U) = l_0 - l_1 U^{-9/2} + \dots \tag{3.12}$$

where  $l_1$  is positive so that the branes come together. For Sakai-Sugimoto without the tachyon,  $l_1 = \frac{2}{3} R_k U_0^4 \sqrt{U_k f_0}$ , where  $f_0 = f(U_0)$ .

Is there a solution in which  $T$  vanishes asymptotically as a power law? Suppose there is such a solution,  $T(U) \sim U^{-\alpha}$ . If  $\alpha > 3/4$  and  $l$  vanishes fast enough, we may once again approximate  $D_T \sim (\frac{U}{R})^{-3/2}$ . As before, we then conclude that  $T$  vanishes exponentially, which contradicts our assumption that  $T$  vanishes as a power law. If  $\alpha < 3/4$  and  $l'$  vanishes fast enough, then we must approximate  $D_T \sim T^2 l_0^2$ . One can see immediately from (3.8) that this also leads to a contradiction. Finally, suppose asymptotically  $l'$  vanishes so slowly that it is the  $l'^2$  term that dominates in  $D_T$  and so we must approximate  $D_T \sim (U/R)^{3/2} l'(U)^2/4$ . Once again it is easy to see from (3.8) that there is no consistent solution. We thus conclude that the only solution in which  $l$  goes to a nonzero constant asymptotically and  $T$  vanishes is the one given by (3.11), (3.12) (after dropping the growing part of  $T$  for large enough  $U$ ).

### 3.2.2 Solution for $U \sim U_0$

Here we are looking for a solution in which  $l \rightarrow 0$  and  $T \rightarrow \infty$  as  $U \rightarrow U_0$ . Let us assume a power law ansatz, namely

$$l(U) \sim (U - U_0)^\alpha, \quad T(U) \sim (U - U_0)^{-\beta}.$$

For a smooth joining of the brane and antibrane at  $U_0$ , the derivative of  $l$  must diverge at this point, which is ensured if  $\alpha < 1$ . Since for this ansatz  $T'^2$  is the largest quantity for  $U \rightarrow U_0$ , we can approximate  $D_T \sim T'(U)^2$ . Moreover, using the asymptotic form of the potential  $V(T) \sim e^{-cT}$  for large  $T$ , we get  $V'(T)/V(T) \sim -c$ . Putting all this in (3.8) we see that the leading term on the l.h.s. is a constant. The first term on the r.h.s. vanishes as a positive power of  $(U - U_0)$ . For consistency with the l.h.s. we then find from the second term on the r.h.s. that (i) if  $\beta > 1$ , we must have  $\beta = 1 + 2\alpha$  and (ii) if  $\beta < 1$ , we must have  $\beta = 1 - 2\alpha$ .  $\beta = 1$  is not allowed since we must have  $0 < \alpha < 1$ . Analyzing equation (3.9)

similarly, we find that in case (i) the l.h.s. of this equation vanishes as a positive power of  $(U - U_0)$ . This is consistent with the r.h.s. only if  $\beta = 2$ , which then gives  $\alpha = 1/2$ . In case (ii) it is the first term on the r.h.s. that vanishes as a positive power of  $(U - U_0)$ . Consistency with the r.h.s. then requires  $\beta = 0$ , which is however inconsistent with our approximations. Hence,  $\alpha = 1/2$ ,  $\beta = 2$  is the only consistent solution we get which has  $l \rightarrow 0$  and  $T \rightarrow \infty$  as  $U \rightarrow U_0$ . This ansatz can now be checked directly and the various coefficients fixed. We get

$$l(U) = \frac{1}{v_1} \sqrt{\frac{26}{U_0 f_0}} \left(\frac{U_0}{R}\right)^{-3/4} (U - U_0)^{1/2} + \dots, \tag{3.13}$$

$$T(U) = \frac{v_1}{4} f_0 \left(\frac{U_0}{R}\right)^{3/2} (U - U_0)^{-2} + \dots, \tag{3.14}$$

where  $v_1$  is a constant which equals the limiting value of  $-V'(T)/V(T)$  as  $T \rightarrow \infty$ . Note that, given the potential, the normalizations of both  $l$  and  $T$  get fixed in terms of  $U_0$ . It is important to mention that this solution exists only for potentials which have the asymptotic behaviour  $V(T) \sim e^{-cT^\gamma}$  for large  $T$ , with  $\gamma < 2$ .<sup>9</sup>

The existence of the solution (3.14), (3.14) shows that tachyon condensation on the flavour brane-antibrane system is intimately connected with chiral symmetry breaking.

For completeness, we note that there exists another solution in which  $T$  does not diverge as  $U \rightarrow U_0$ . Let us assume that  $T$  goes to a nonzero constant as  $U \rightarrow U_0$ . In this case we can approximate  $D_T \sim f(U)(U/R)^{3/2}l'(U)^2/4$ . Substituting in (3.8) we see that the l.h.s. diverges as  $(U - U_0)^{-\alpha}$ . The first term on the r.h.s. vanishes as a positive power, but the second term diverges as  $(U - U_0)^{\alpha-1}$ . For consistency we must have  $\alpha = 1/2$ . The resulting solution

$$l(U) = \frac{4}{U_0} \left(\frac{R^3}{f_0(5f_0 + 3)}\right)^{1/2} (U - U_0)^{1/2} + \dots, \tag{3.15}$$

$$T(U) = t_0 + \frac{2}{(5f_0 + 3)} \left(\frac{R^3}{U_0}\right)^{1/2} \frac{V'(U_0)}{V(U_0)} (U - U_0) + \dots \tag{3.16}$$

also satisfies (3.9). Note that no special condition was required for the tachyon potential to get this solution; this solution exists for any potential.

To get a complete solution, one needs to use numerical tools since the equations cannot be solved analytically. The numerical calculations are in progress and will be reported in a forthcoming longer version of this work [42].

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<sup>9</sup>This condition is not satisfied by the potential obtained by a boundary string field theory computation [36–39] for which  $\gamma = 2$ . This is not necessarily a contradiction and probably indicates a nontrivial field redefinition that relates fields we are using here to those used in the boundary string field theory. A similar observation has been made earlier in connection with the tachyon kink and vortex solutions on the brane-antibrane system in [31]. Note, however, that a calculation of S-matrix elements of tachyons and gauge fields reported in [41] seems to favour the boundary string field theory potential.

#### 4. The meson spectra

In this section we will discuss the spectra for various low spin mesons which are described by the fluctuations of the flavour branes around the classical solution. The action for the fluctuations of the gauge fields can be computed from (3.1). Parametrizing the complex tachyon  $\tau$  in terms of its magnitude and phase,  $\tau = T e^{i\theta}$ , we get

$$\Delta S_{\text{gauge}} = - \int d^4x dU \left[ a(U) A_U^2 + b(U) A_\mu^2 + c(U) ((F_{\mu\nu}^V)^2 + (F_{\mu\nu}^A)^2) + e(U) F_{\mu U}^A A^\mu + d(U) ((F_{\mu U}^V)^2 + (F_{\mu U}^A)^2) \right], \quad (4.1)$$

$$a(U) = V_4 V(T) U^4 \left( \frac{U}{R} \right)^{-3/4} \frac{T^2}{\sqrt{D_T}}, \quad (4.2)$$

$$b(U) = V_4 V(T) U^4 \left( \frac{U}{R} \right)^{-3/4} \sqrt{D_T} \left( \frac{U}{R} \right)^{-3/2} \frac{T^2}{Q} \left( 1 + \frac{f^2 T^2 l^2 l'^2}{4 D_T} \left( \frac{U}{R} \right)^3 \right), \quad (4.3)$$

$$c(U) = V_4 V(T) U^4 \left( \frac{U}{R} \right)^{-3/4} \sqrt{D_T} \frac{1}{8} \left( \frac{U}{R} \right)^{-3}, \quad (4.4)$$

$$d(U) = V_4 V(T) U^4 \left( \frac{U}{R} \right)^{-3/4} \left( \frac{U}{R} \right)^{-3/2} \frac{Q}{4 \sqrt{D_T}}, \quad (4.5)$$

$$e(U) = V_4 V(T) U^4 \left( \frac{U}{R} \right)^{-3/4} \frac{f T^2 l l'}{2 \sqrt{D_T}}. \quad (4.6)$$

Here  $F^V$  is the field strength for the vector gauge field  $V = (A_1 + A_2)$  and  $F^A$  is the field strength for the gauge-invariant combination of the axial vector field and the phase of the tachyon,  $A = (A_1 - A_2 - \partial\theta)$ .

The gauge field  $V_\mu(x, U)$  gives rise to a tower of vector mesons while the fields  $A_\mu(x, U)$  and  $A(x, U)$ , which are gauge invariant, give rise to towers of axial and pseudoscalar mesons. Notice that the coefficients  $a(U)$ ,  $b(U)$  and  $e(U)$  vanish if the tachyon is set to zero. In the absence of the tachyon the vector and axial vector mesons acquire masses because of a nonzero  $d(U)$ , but there is always a massless ‘‘pion’’.<sup>10</sup> The presence of the tachyon is thus essential to give a mass to the ‘‘pion’’. Also note that with the tachyon present, the masses of the vector and axial vector mesons are in principle different.

In the following, we will be using the gauge  $V_U = 0$ . As we have already noted,  $A_U$  is gauge invariant. Expanding in modes, we have

$$\begin{aligned} V_\mu(x, U) &= \sum_m V_\mu^{(m)}(x) W_m(U) \\ A_\mu(x, U) &= \sum_m A_\mu^{(m)}(x) P_m(U), \\ A_U(x, U) &= \sum_m \phi^{(m)}(x) S_m(U), \end{aligned} \quad (4.7)$$

<sup>10</sup>Strictly speaking, for the U(1) case under discussion, this pseudoscalar is the  $\eta'$ . It is massless here because of the  $N_c \rightarrow \infty$  limit in which we are working.

where  $\{W_m(U)\}$ ,  $\{P_m(U)\}$ s and  $\{S_m(U)\}$  form complete sets of basis functions. The fields  $\{V_\mu^{(m)}\}$ ,  $\{A_\mu^{(m)}\}$  and  $\{\phi^{(m)}\}$  form towers of vector, axial-vector and pseudoscalar mesons in the physical  $(3+1)$ -dimensional space-time. Note that  $\partial^\mu A_\mu^{(m)}$  and  $\phi^{(m)}$  mix. After suitably shifting  $A_\mu^{(m)}$  by an appropriate linear combination of  $\partial_\mu \phi^{(m)}$ s, the mixing can be removed. The spectrum may then be read off from the quadratic action

$$\Delta S_{\text{gauge}} = - \int d^4x \sum_m \left[ \frac{1}{4} F_{\mu\nu}^{V(m)} F^{V(m)\mu\nu} + \frac{1}{2} \lambda_m^V V_\mu^{(m)} V^{(m)\mu} + \frac{1}{4} F_{\mu\nu}^{A(m)} F^{A(m)\mu\nu} + \frac{1}{2} \lambda_m^A A_\mu^{(m)} A^{(m)\mu} + \frac{1}{2} \partial_\mu \phi^{(m)} \partial^\mu \phi^{(m)} + \frac{1}{2} \lambda_m^\phi \phi^{(m)} \phi^{(m)} \right], \quad (4.8)$$

where in the vector and axial vector sectors we have imposed the orthonormality conditions

$$\int dU c(U) P_m(U) P_n(U) = \frac{1}{4} \delta_{mn} = \int dU c(U) W_m(U) W_n(U), \quad (4.9)$$

and the eigenvalue equations

$$\begin{aligned} - (d(U) W'_m(U))' &= 2\lambda_m^V c(U) W_m, \\ - (d(U) P'_m(U))' + \left( b(U) + \frac{1}{2} e'(U) \right) P_m(U) &= 2\lambda_m^A c(U) P_m(U). \end{aligned} \quad (4.10)$$

In the pseudoscalar sector, we need the conditions

$$\begin{aligned} \int dU a(U) S_m(U) S_n(U) &= \frac{1}{2} \lambda_m^\phi \delta_{mn} \\ \left( K - \frac{1}{2} J^T L^{-1} J \right)_{mn} &= \frac{1}{2} \delta_{mn}, \end{aligned} \quad (4.11)$$

where

$$\begin{aligned} J_{mn} &= \int dU [e(U) P_m(U) - 2d(U) P'_m(U)] S_n(U), \\ K_{mn} &= \int dU d(U) S_m(U) S_n(U), \\ L_{mn} &= \lambda_m^A \delta_{mn} \end{aligned} \quad (4.12)$$

One can also consider fluctuations in  $T$  and  $l$ . There is mixing in this sector also. These fluctuations give rise to towers of scalars whose masses depend on the background value of the tachyon. We defer details of these calculations to a forthcoming publication [42].

#### 4.1 Quark mass and chiral condensate

In this section we will give evidence for the identifications made below (3.11) for the parameters  $T_\pm$  with the chiral condensate and quark mass. We note that for  $T(U) = 0$ ,  $a(U)$  vanishes and hence  $\lambda_m^\phi$ , given by the first of (4.11), also vanishes. We see once again that a nonzero tachyon is required for nonzero pseudoscalar masses. Furthermore, since  $V(T)$  vanishes exponentially for large  $T$ , the region of  $U$  in which  $T$  is small, but not too small, dominates the integral in (4.11). This is the intermediate region discussed

below (3.11). In this region  $T$  can be essentially replaced by (3.11). Consider the lightest mass state. For this state, we have

$$\frac{1}{2}\lambda_0^\phi = \int dU a(U)(S_0(U))^2 \tag{4.13}$$

The r.h.s. of this equation involves the quantity  $a(U)$  which is proportional to  $T^2$ . Using (3.11) and retaining to lowest order in the quark mass parameter, which we have identified with  $T_-$ , we see that this gives  $\lambda_0^\phi \sim T_- T_+$ .<sup>11</sup> This firms up the identification of  $T_+$  with the chiral condensate. This relation is then essentially the Gell-Mann-Oakes-Renner relation.

## 5. Discussion

In this paper we have proposed a modified Sakai-Sugimoto model which includes the open string tachyon stretching between the flavour branes and antibranes. Taking the tachyon into account is essential for the consistency of the setup. Our modification preserves the nice geometric picture of chiral symmetry breaking of the Sakai-Sugimoto model and at the same time relates chiral symmetry breaking to tachyon condensation; the tachyon becomes infinitely large in the infrared region where the joining of the flavour branes signals chiral symmetry breaking.

We have shown that the tachyon condensate is essential to give the goldstone bosons nonzero masses. We have identified parameters in the tachyon field profile which correspond to the quark bare mass and chiral condensate. We also briefly discussed different types of low spin meson fluctuations. A more complete discussion with numerical estimates for masses etc is under preparation.

There are several directions in which the present ideas can be extended. It would be interesting to discuss this model at finite temperature and describe the chiral symmetry restoration transition and study the phase diagram in some detail. The connection with tachyon condensation seems fascinating and a deeper understanding would be useful. Finally, baryons have been discussed in the Sakai-Sugimoto model. It turns out that they have a very small size. This may change in the presence of the tachyon. This is because in the presence of the tachyon, the flavour energy momentum tensor is concentrated far away from the infrared region where the branes meet. It would be very interesting to investigate whether this effect actually makes a difference to the baryon size.

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<sup>11</sup>To get this result, one first does the calculation with a given cut-off,  $U_{\max}$ , with the condition  $T_+ e^{-l_0 U_{\max}} \gg T_- e^{l_0 U_{\max}}$ . This is the condition that  $T_-$  is small and justifies retaining upto linear in  $T_-$  term only. One must then remove the cut-off,  $U_{\max} \rightarrow \infty$ , keeping the condensate and the physical quark mass fixed. This should remove the  $T_+^2$  term, consistent with the fact that for  $T_- = 0$  the pion mass must vanish.

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